Proposed problem

Let $\{a_k\}_{k\geq 1}$ *be a monotonic sequence of real positive numbers such that* $\sum_{n=1}^{\infty} a_n < \infty$. *Moreover* $\{a_k\}$ *fulfills the conditions*

$$2^{-n+1}a_{2^{n+1}} \geq a_k - a_{k+1} \geq 2^{-n}a_{2^{n+1}}, \quad \forall\ 2^n \leq k \leq 2^{n+1}-1, \quad \forall\ k \geq 1$$

Let α *be a quadratic irrational. Prove that the following quantity is bounded,*

$$\frac{1}{\ln n}\sum_{k=1}^{n}\frac{a_k}{\sin(k\pi\alpha)}, \qquad n \geq 1$$

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Best regards
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